International Journal of Theoretical Physics, Vol. 39, No. 7, 2000

Cosmological Perturbations of Quantum Mechanical Origin: Are Nonvacuum Initial States Allowed?

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Received May 17, 2000

We address the question of whether nonvacuum initial states for cosmological perturbations are allowed, or whether they are ruled out on the basis of present experimental and observational data. Our choice of a nonvacuum initial state is guided by the idea that the initial state could have a built-in characteristic scale. We find that a class of initial states can fit the data; however, the initial states must be close to the vacuum.

1. INTRODUCTION

The inflationary paradigm was proposed in order to explain the shortcomings of the Big Bang cosmological model. In addition, it offers a scenario for the generation of the primordial density perturbations, which can lead to the formation of the observed large-scale structure. Generic predictions of simple inflationary models are a scale-invariant spectrum with, provided the quantum fields are initially placed in the vacuum, Gaussian fluctuations.

Indeed, it is almost [1] always assumed that the initial state of the perturbations is the vacuum. Let us see whether this assumption can be justified. The choice of the initial quantum state in which the quantum fields are place should be made on the basis of full quantum gravity. This theory is at present unknown. One may select a "maximally symmetric state" [2] as an initial state of the universe. However, in the context of quantum gravity, there exists a privileged scale, the Planck scale. Thus, we believe that there is no theoretical proof for taking the vacuum as the initial state in which quantum fields are placed. Consequently, we find [3] that it is worth studying

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0020-7748/00/0700-1841$18.00/0

nonvacuum initial states for cosmological perturbations. Our choice of a nonvacuum initial state is guided by the idea that the initial state could have a built-in characteristic scale. Rather than relying on theoretical arguments, we allow for the possibility of nonvacuum initial states and we examine [3] whether the consequences for the cosmic microwave background radiation (CMBR) anisotropies and the power spectrum of galaxies and cluster of galaxies are in conflict with the current data. We will find [3] that there exists a window for the free parameters of the model, which fits the observational data.

2. NONVACUUM INITIAL STATE FOR COSMOLOGICAL PERTURBATIONS

2.1. Perturbations of Quantum Mechanical Origin

The background model is described by a Friedmann–Lemaître–Robertson–Walker (FLRW) metric whose spacelike sections are flat. Inflation is driven by a single scalar field, $\varphi_0(\eta)$. We introduce the background quantities $\mathcal{H}(\eta) \equiv a'/a$ and $\gamma(\eta) \equiv 1 - (\mathcal{H}^1/\mathcal{H}^2)$ (the primes denote the derivatives with respect to conformal time η). In the synchronous gauge, the line element for the FLRW background plus scalar perturbations reads [4]

$$ds^2 = a^2(\eta)\left\{-d\eta^2 + \left[\delta_{ij} + \frac{1}{(2\pi)^{3/2}}\int d\mathbf{k}\left(h(\eta,\mathbf{k})\delta_{ij} - \frac{h_l(\eta,\mathbf{k})}{k^2}k_ik_j\right)e^{i\mathbf{k}\cdot\mathbf{x}}\right]dx^i\,dx^j\right\} \tag{1}$$

where the functions h, h_l represent the scalar perturbations of the gravitational field and the longitudinal–longitudinal perturbation, respectively. The perturbations of the scalar field are Fourier decomposed according to

$$\delta\varphi(\eta,\mathbf{x}) = \frac{1}{(2\pi)^{3/2}}\int d\mathbf{k}\;\varphi_1(\eta,\mathbf{k})e^{i\mathbf{k}\cdot\mathbf{x}} \tag{2}$$

The perturbed Einstein equations couple the scalar sector, h and h_l, to the perturbed scalar field φ_1. The residual gauge-invariant quantity $\mu(\eta, \mathbf{k})$ defined by [4]

$$\mu \equiv \frac{a}{\mathcal{H}\sqrt{\gamma}}(h' + \mathcal{H}\gamma h) \tag{3}$$

can be used to express all other relevant quantities. In the synchronous gauge, $\mu(\eta, \mathbf{k})$ is related to the gauge-invariant Bardeen potential through the equation [5]

$$\Phi = \frac{\mathcal{H}\gamma}{2k^2}\left(\frac{\mu}{a\sqrt{\gamma}}\right)' \tag{4}$$

The perturbed Einstein equations imply that the equation of motion for $\mu(\eta, \mathbf{k})$ is [4]

$$\mu'' + \left[k^2 - \frac{(a\sqrt{\gamma})''}{(a\sqrt{\gamma})}\right]\mu = 0 \tag{5}$$

Since the origin of the perturbations is quantum mechanical, the normalization of the perturbed scalar field, and consequently of the scalar perturbations, is completely fixed in the high-frequency regime. In this regime, the perturbed field can be considered as a free field in the curved FLRW background space-time. The Fourier component operator of the perturbed field $\hat{\varphi}_1(\eta, \mathbf{k})$ in the limit $k \to +\infty$ is

$$\hat{\varphi}_1(\eta, \mathbf{k}) = \frac{\sqrt{\hbar}}{a(\eta)}\left[c_{\mathbf{k}}(\eta_0)\frac{e^{-ik(\eta-\eta_0)}}{\sqrt{2k}} + c^{\dagger}_{-\mathbf{k}}(\eta_0)\frac{e^{-ik(\eta-\eta_0)}}{\sqrt{2k}}\right] \tag{6}$$

In the high-frequency regime

$$\lim_{k\to+\infty} \hat{\mu}(\eta, \mathbf{k}) = -4\sqrt{\pi G}a(\eta) \lim_{k\to+\infty} \hat{\varphi}_1(\eta, \mathbf{k}) \tag{7}$$

If we define

$$f_k(\eta) \equiv -4\sqrt{\pi}[(\mathcal{H}\gamma)/(2k^2)][\xi_k/(a\sqrt{\gamma})'] \tag{8}$$

where $\xi_k(\eta)$ is the solution of the equation of motion for μ such that $\lim_{k\to+\infty} \xi_k = e^{-ik(\eta-\eta_0)}/\sqrt{2k}$, we find that the dimensionless Bardeen operator $\hat{\Phi}(\eta, \mathbf{x})$ reads [3]

$$\hat{\Phi}(\eta, \mathbf{x}) = \frac{l_{\mathrm{Pl}}}{(2\pi)^{3/2}}\int d\mathbf{k}\,[c_{\mathbf{k}}(\eta_0)f_k(\eta)e^{i\mathbf{k}\cdot\mathbf{x}} + c^{\dagger}_{\mathbf{k}}(\eta_0)f^{*}_k(\eta)e^{-i\mathbf{k}\cdot\mathbf{x}}] \tag{9}$$

2.2. Quantum States

We specify the quantum state in which $\hat{\Phi}(\eta, \mathbf{x})$ is placed, under the hypothesis that the perturbations are initially in a nonvacuum state. Our choice of nonvacuum states is guided by the idea that one must introduce a scale in the theory, denoted by the wave number k_0. We examine three different nonvacuum state [3].

Let $\mathcal{D}$ be the domain in momentum space between the spheres of radius $k_0 - \sigma$ and $k_0 + \sigma$. Since $\mathcal{D}$ is invariant under rotations, this definition is compatible with the assumption of isotropy of the universe. The first state we consider is [3]

$$\left|\Psi_1(k_0, \sigma, n)\right\rangle = \bigotimes_{\mathbf{k}\in\mathcal{D}(k_0,\sigma)} \left|n_{\mathbf{k}}\right\rangle \bigotimes_{\mathbf{P}\notin\mathcal{D}(k_0,\sigma)} \left|0_{\mathbf{P}}\right\rangle \tag{10}$$

The state $|n_{\mathbf{k}}\rangle$ is an n-particle state satisfying, at $\eta = \eta_0$, $c_{\mathbf{k}}|n_{\mathbf{k}}\rangle = \sqrt{n}|(n - 1)_{\mathbf{k}}\rangle$ and $c_{\mathbf{k}}^{\dagger}|n_{\mathbf{k}}\rangle = \sqrt{n+1}|(n+1)_{\mathbf{k}}\rangle$. Clearly, in the state $|\Psi_1\rangle$ the transition between the empty and the filled modes is sharp and therefore rather unphysical. In order to "smooth out" the quantum state $|\Psi_1\rangle$, we introduce the function $g(\sigma)$. We thus define the state [3]

$$\left|\Psi_2(k_0, n)\right\rangle \equiv \int d\sigma g(\sigma)\left|\Psi_1(k_0, \sigma, n)\right\rangle \tag{11}$$

where, *a priori*, $g(\sigma)$ is an arbitrary function of σ.

Finally we define a state that will allow us to work with an effective number of quanta which will no longer be an integer. This state is [3]

$$\left|\Psi_3(k_0)\right\rangle \equiv \sum_{n=0}^{\infty} h(n)\left|\Psi_2(k_0, n)\right\rangle \tag{12}$$

The function $h(n)$ is arbitrary. The state $|\Psi_3(k_0)\rangle$, which depends on k_0 and on the free parameters characterizing the functions $g(\sigma)$ and $h(n)$, seems to be the most natural rotational-invariant, smooth, quantum state that privileges a scale.

2.3. Power Spectra

The power spectrum of $\hat{\Phi}(\eta, \mathbf{x})$ in the state $|\Psi\rangle$, denoted by $P_{\Phi}(k; |\Psi\rangle)$, is defined through the calculation of the two-point correlation function $K_2(r; |\Psi\rangle)$:

$$K_2(r; |\Psi\rangle) = \int_0^{\infty} \frac{dk}{k} \frac{\sin kr}{kr} k^3 P_{\Phi}(k; |\Psi\rangle) \tag{13}$$

We calculate the power spectra of the Bardeen potential operator in the three previously defined states $|\Psi_i\rangle$, $i = 1,2,3$. For the state $|\Psi_1\rangle$, we obtain [3]

$$k^3 P_{\Phi}(k; |\Psi_1\rangle) = \frac{l_{\mathrm{Pl}}^2}{2\pi^2} k^3 |f_k|^2 \{1 + 2n[H(k - k_0 + \sigma) - H(k - k_0 - \sigma)]\} \tag{14}$$

where H is a Heaviside function. This spectrum is not continuous, due to the very crude definition of the state $|\Psi_1\rangle$. For the quantum state $|\Psi_2\rangle$, we obtain [3]

$$k^3 P_{\Phi}(k; |\Psi_2\rangle) = \frac{l_{\mathrm{Pl}}^2}{2\pi^2} k^3 |f_k|^2 (1 + 2n e^{-(k-k_0)^2/\Sigma^2}) \tag{15}$$

where n is an integer and Σ is a free parameter.

Finally, for the state $|\Psi_3\rangle$, we get [3]

$$k^3 P_\Phi(k; |\Psi_3\rangle) = \frac{l_{\rm Pl}^2}{2\pi^2} k^3 |f_k|^2 (1 + 2n_{\rm eff} e^{-(k-k_0)^2/\Sigma^2}) \tag{16}$$

Here the effective number of quanta $n_{\rm eff}$ is given by [3]

$$n_{\rm eff} = \frac{\Sigma_{n=0}^{\infty} n|h(n)|^2}{\Sigma_{n=0}^{\infty} |h(n)|^2} \tag{17}$$

The spectra of Eqs. (15) and (16) possess a peak around the scale k_0. The position of the peak is controlled by the value of k_0, its width by Σ, and its height by n or $n_{\rm eff}$.

We need the primordial spectrum only for large wavelengths and in this limit,

$$k^3 P_\Phi(k; |0\rangle) = A_{\rm S} k^{n_{\rm S} - 1} \tag{18}$$

with

$$A_{\rm S} = \frac{l_{\rm Pl}^2}{l_0^2} \frac{\gamma(1 + \beta)^2}{2^{2\beta+4} \cos^2(\beta\pi)\Gamma^2(\beta + 5/2)} \tag{19}$$

where

$$n_{\rm S} = 2\beta + 5, \qquad a(\eta) = l_0|\eta|^{1+\beta} \quad (\beta \lesssim -2) \tag{20}$$

The initial power spectrum for the Bardeen operator placed in the state $|\Psi_2\rangle$ reads [3]

$$k^3 P_\Phi(k; |\Psi_2\rangle) = A_{\rm S} k^{n_{\rm S}-1} (1 + 2n e^{-(k-k_0)^2/\Sigma^2}) \tag{21}$$

If the state is $|\Psi_3\rangle$, we just have to replace the integer n with the real number $n_{\rm eff}$.

There is an interesting prediction of our model concerning the statistics of the fluctuations. More precisely, in our model the four-point correlation function shows deviations from Gaussianity. On the other hand, broken scale-invariant spectra, where the privileged scale arises as a privileged energy in the inflaton potential, lead to Gaussian fluctuations [6]. Recently, some non-Gaussianity was detected [7–9] in the CMBR map. If it is indeed confirmed, then broken scale-invariant models, our model, as well as standard inflation are ruled out. However, if some non-Gaussianity is present in the CMBR at the level of the four-point correlation function, then our model could account for this.

3. COMPARISON WITH OBSERVATIONS

Observations of the CMBR anisotropies and of the matter power spectrum will give the best values of k_0, Σ, and $n/n_{\rm eff}$. We take the following values of the cosmological parameters: the Hubble parameter is $h = 0.5$, the baryonic matter-density parameter is $\Omega_b = 0.05$, the density parameter $\Omega_0 = 1$ and, there is no significant reionization. We consider two models: (i) the SCDM model with Λ-density parameter $\Omega_\Lambda = 0$ and CDM-density parameter $\Omega_c = 0.95$ and (ii) the ΛCDM model with $\Omega_\Lambda = 0.6$, $\Omega_c = 0.35$.

The spectrum must be normalized, in other words, the value of $A_{\rm S}$ must be determined. To do so, we use the value of $Q_{\text{rms-PS}} \sim 18$ μK measured by the COBE satellite. In the large-wavelength approximation, we have $\delta T/T \sim (1/3)\Phi$. The multipole can be written as [3]

$$C_l = \frac{4\pi}{9}\int_0^{+\infty}\frac{dk}{k}\,[j_l[k(\eta_0 - \eta_{\rm LSS})]^2 A_S(n_S)k^{n_S-1}(1 + 2ne^{-(k-k_0)^2/\Sigma^2})] \tag{22}$$

where j_l is a spherical Bessel function of order l, and η_0 and $\eta_{\rm LSS}$ denote respectively the conformal times now and at the last scattering surface. For a scale-invariant spectrum we obtain [3]

$$A_{\rm S} = \frac{9.4\times 10^{-10}}{1 + 24nI}, \qquad \text{where} \quad I \equiv \int_0^{+\infty}\frac{du}{u^{2-n_{\rm S}}}\,[j_2(u)]^2 e^{-(u-u_0)^2/U^2} \tag{23}$$

As a next step, we must choose the three parameters k_0, n, and Σ. The power spectrum seems [10] to contain large-amplitude features at the scale $\approx 100h^{-1}$ Mpc, which corresponds to a wave number equal to $0.062h$ Mpc^{-1}. Since no other value for a privileged scale has been detected so far, we choose [3]

$$k_0 = 0.031\ \text{Mpc}^{-1} \tag{24}$$

The exact value of Σ, as long as it is not too large, will not affect our conclusions. In what follows, we consider [3]

$$\Sigma = 0.3k_0 \tag{25}$$

Let us discuss the matter power spectrum. The baryon power spectrum is

$$\frac{\delta\rho_{\rm b}}{\rho_{\rm b}} = AT^2(k)\,\frac{g^2(\Omega_0)}{g^2(\Omega_m)}\,k[1 + 2ne^{-(k-k_0)^2/\Sigma^2}] \tag{26}$$

where $T(k)$ is the transfer function and the coefficient A is obtained by normalizing the spectrum to COBE data. This leads to

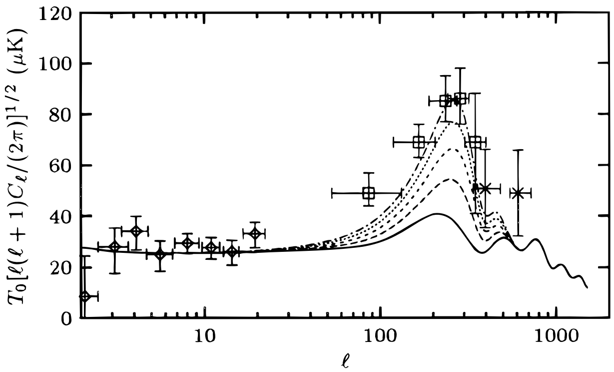

Fig. 1. CMBR anisotropies for the SCDM model, with $n_{\rm eff}$ ranging from 0 to 4 with a step of 1 (from the bottom to the top). Diamonds represent COBE data, squares the Saskatoon data, and crosses the CAT data.

$$A = (2l_{\rm H})^4 \frac{6\pi^2}{5} \frac{Q^2_{\text{rms-PS}}}{T_0^2} \frac{1}{1 + 24nI} = \frac{6.82 \times 10^5}{1 + 24nI} h^{-4} \,\text{Mpc}^4 \qquad (27)$$

where the Hubble radius $l_{\rm H}$ is equal to $3000h^{-1}$ Mpc.

We plot the multipole moments and the power spectrum for different values of n and/or $n_{\rm eff}$. The theoretical predictions for the multipole moments and the power spectra are obtained using a Boltzmann code.

We consider the sum of the scalar plus the tensor contributions to the CMBR anisotropies. In Figs. 1 and 2 we show the CMBR anisotropies and the matter power spectrum for the SCDM model, including both scalar and tensor contributions. In Figs. 3 and 4 we show the CMBR anisotropies and the matter power spectrum for the ΛCDM model including both scalar and

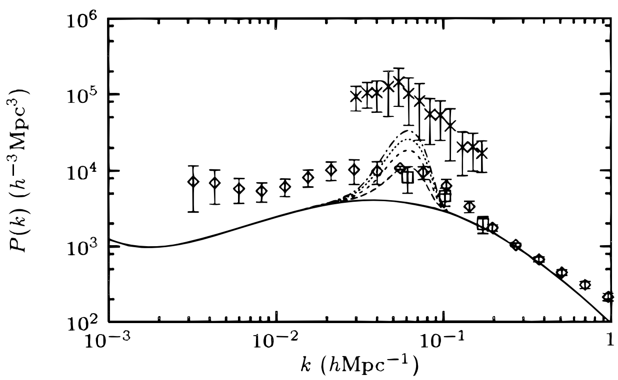

Fig. 2. Power spectrum for the SCDM model, with $n_{\rm eff}$ ranging from 0 to 4 with step of 1. Diamonds represent the APM data, squares the velocity field measurements, and crosses the data given by Einasto *et al.*

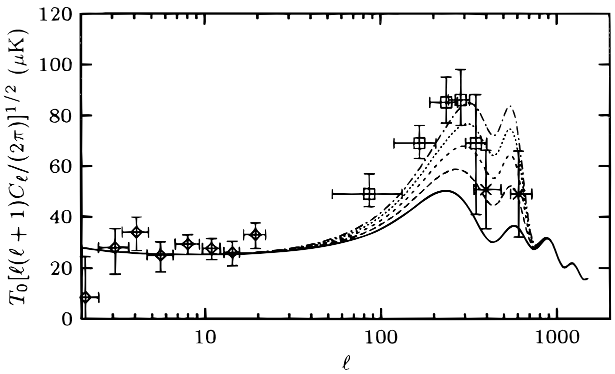

Fig. 3. Same as Fig. 1, but for the ΛCDM model, with n_{eff} ranging from 0 to 4 with a step of 1 (from the top to the bottom).

tensor contributions. Clearly, matter power spectrum data favor a higher value of n_{eff} than CMBR anisotropy data.

For both types of models, with and without a cosmological constant, CMBR anisotropies measurements require [3] a higher value of n_{eff} than in the case of an absence of tensor mode contribution. This is in agreement with the matter power spectra.

4. CONCLUSIONS

We examined whether current experimental and observational data allow nonvacuum initial states for cosmological perturbations. Our choice of a nonvacuum initial state was guided by the idea that the initial state could have a built-in characteristic scale. We calculated the power spectra of the

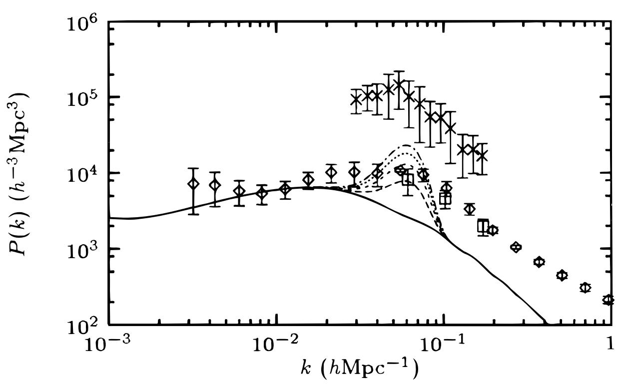

Fig. 4. Same as Fig. 2, but for the ΛCDM model, with n_{eff} ranging from 0 to 4 with a step of 1 (from the top to the bottom).

Bardeen potential and compared their theoretical predictions with the CMBR anisotropy measurements and the redshift surveys of the distribution of galaxies.

There exists a window for the free parameters such that good agreement between the data and the theoretical predictions is possible. However, to account for the observations, the initial state cannot be too different from the vacuum.

The generic predictions of our model are a high amplitude of the first acoustic peak, a nontrivial feature in the matter power spectrum, and deviations from Gaussianity in the CMBR map. More experimental and observational data are needed to determine whether our new class of models represents a viable alternative to the standard theory.

ACKNOWLEDGMENTS

It is a pleasure to thank Edgard Gunzig for inviting me to Peyresq and for the wonderful atmosphere during the whole meeting. I would also like to thank Jérôme Martin and Alain Riazuelo with whom I collaborated on this work.

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